Distributed Control and Generation Estimation Method for Integrating High-Density Photovoltaic Systems

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Abstract—The presence of distributed generators (DGs) such as photovoltaic systems (PVs) is increasing significantly in distribution networks, and in order to accommodate a higher penetration of DGs, technical issues arising from fluctuation and unpredictability of their power output must be addressed. It is beneficial if DGs of high penetration can be dispatched when necessary. To this end, a distributed control and generation estimation approach is developed to dispatch multiple DGs, each of which consists of a PV and a controllable load. A strongly connected digraph with a row stochastic adjacency matrix is sufficient for the communication topology. A distributed weights adjustment algorithm adaptively makes the adjacency matrix doubly stochastic so that the aggregated power generation capacity can be estimated. Then, the expected consensus operational point of the DGs is calculated by those DGs that can obtain power dispatch command from the supervisory control and data acquisition system and is propagated to the rest of the DGs with a consensus algorithm. With this method, all the DGs operate at the same ratio of available power, while their aggregated power meets the power dispatch command. Simulations in the IEEE standard 34-bus distribution network verify the effectiveness of the proposed approach.

Index Terms—Consensus, distributed estimation, doubly stochastic, photovoltaic (PV), power dispatch.

I. INTRODUCTION

Due to increasing concern about the environment, skyrocketing cost for fossil fuels, and desire to increase the diversity in energy supply, the focus of electricity industry is gradually moving from conventional energy resources (e.g., fossil fuels) to distributed energy resources in the past decades. Among all forms of distributed energy resources, photovoltaic systems (PVs) are currently being widely installed worldwide and have a promising prospect [1].

However, since the installed PV capacity is not uniformly distributed, certain regions in the world are already experiencing a very high local PV penetration [2]. Owing to the fluctuation and unpredictability of PV power output, serious problems may arise, such as voltage variation [2] or even voltage collapse [3]. Also, because synchronous generators are partly replaced by PVs, the frequency regulation capability and system inertia are drastically decreasing, which could lead to severe frequency fluctuation under certain disturbances.

Considerable research has been done to resolve these problems by managing PV power output. For example, it is proposed in [4] that power fluctuation of PVs be reduced by directly curtailing their power output and installing controllable loads to dissipate the excess power and that this approach can result in lower revenue loss than installing energy storage systems in certain occasions. The concept of virtual energy storage is introduced and studied in [5] and [6] to consider multiple distributed generators (DGs) and other controllable elements as a whole and to regulate their aggregated power output as a traditional generating unit. This idea enables DGs to participate in generation scheduling and provide ancillary services to the power grid, e.g., frequency regulation and voltage stabilization [7], [8].

However, most of the above works concern centralized or decentralized control only. In detail, centralized control schemes (see [9], [10] and the references therein) calculate the dispatch commands based on the optimal power flow strategy and send them to all the DGs through the communication channels. These schemes require global information to find the operational point, thus are not robust to communication interruptions or topology changes. Decentralized control methods, such as maximum power point tracking (MPPT) and frequency droop control (see [7] and [11]–[14]) require local information only. Therefore, they are robust and save the cost of communication network. However, as the capacity of DGs increase, it becomes difficult to coordinate all the DGs to serve a common purpose.

In comparison, a distributed control scheme, which only requires local communication network and information exchange, can enjoy the advantage of economic efficiency and robustness and meet the requirement of plug and play. Its basic idea is to

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use an iterative way to obtain the results that can be obtained by centralized control.

In the last two decades, distributed control has been extensively studied in many domains, e.g., cooperative control of unmanned vehicles, etc [15]. It has emerged as a challenging new area in the research of smart grid recently. Specifically, a distributed control scheme is formulated in [16] to control the power output of multiple PVs, so that some ancillary services can be provided, e.g., regulating the active power flow across certain transmission line. However, this method can only be applied to some special distribution networks, e.g., radial power networks, otherwise global information is required to find the aggregated power. In [17], a distributed control is proposed to solve economic dispatch problem, which estimates the mismatch between the demand and total amount of power generation and uses it as a feedback mechanism to iteratively drive each generator to its optimal operational point. This control design only requires directed communication network, but global information of the communication topology is necessary to initialize the column stochastic matrix as well as determine the control parameters; thus, this method is not completely distributed.

In this paper, a distributed control and generation estimation (DCAGE) approach is presented with the following features.

1. It works with the existing supervisory control and data acquisition (SCADA) system at the substation level by accepting aggregated power dispatch signals.

2. It builds upon directed communication networks that facilitate information exchanges among the DGs.

3. While there is no global information in a distribution network, it enables dynamic estimation of the aggregated generation capacity of the DGs.

4. It accomplishes the aggregated power dispatch by distributively controlling the power output of the DGs, so that all of them are serviced equitably.

In comparison with existing literature, the proposed DCAGE approach is fully distributed and has several distinctions. It can be viewed as a distributed implementation of the centralized control scheme and thus incorporates its main advantage: since this method is based on the estimation of aggregated generation capacity, it can be made ultimately fast (which is only limited by the protocol of the communication infrastructure). In addition, the distributed weights adjustment algorithm is designed to adaptively make the adjacency matrix related to a strongly connected digraph doubly stochastic, based on which lots of distributed algorithms can be generalized from undirected graph to digraph, such as average consensus [18], [19].

The remaining of this paper is organized as follows. A brief review of basic knowledge on graph theory and a distributed estimator of the first left eigenvector are presented in Section II. The DCAGE problem is formulated in Section III and a centralized solution is provided for the case that the global information is available. The proposed DCAGE approach is systematically developed in Section IV. Simulation results are included in Section V to demonstrate effectiveness of the proposed method. And conclusions are drawn in Section VI.

II. PRELIMINARIES

This section introduces the basic results of graph theory and distributed estimation of the first left eigenvector, both of which are fundamental to the proposed DCAGE approach.

A. Graph Theory

Consider a digraph \( D = (\mathcal{V}, \mathcal{E}) \), where \( \mathcal{V} = \{1, 2, \ldots, N\} \) and \( \mathcal{E} \subset \mathcal{V} \times \mathcal{V} \) denote the sets of nodes and directed paths, respectively. Node \( j \) is said to be adjacent to node \( i \) if there exists a directed path \((j, i) \in \mathcal{E} \), with \( i \) being the head and \( j \) being the tail. Analogously, neighborhood set \( N_i \subseteq \mathcal{V} \) of node \( i \) is \( \{k \in \mathcal{V} | (k, i) \in \mathcal{E}\} \). Without loss of any generality, the adjacency matrix \( A(D) \in \mathbb{R}^{N \times N} \) considered is weighted as

\[
[A(D)]_{ik} = \begin{cases} 
  a_{ik} > 0, & \text{if } (k, i) \in \mathcal{E} \text{ and } k \neq i \\
  1 - \sum_{j \in N_i} a_{ij}, & \text{if } k = i \\
  0, & \text{otherwise}
\end{cases}
\]

i.e., \( A(D) \) is designed to be nonnegative and row stochastic (each of its rows sums to 1). Matrix \( A(D) \) is column stochastic if each of its columns sums to 1. It is doubly stochastic if it is both row and column stochastic.

Moreover, if there is a directed path between any pair of nodes or, equivalently, there is one globally reachable node in \( D \) it is called connected. If there is a directed path from \( i \) to \( k \) and a directed path from \( k \) to \( i \) for every pair of nodes \( i, k \in \mathcal{V} \), it is called strongly connected, and the corresponding \( A(D) \) is irreducible. Since \( A(D) \) is row stochastic, \( \lambda_1 = 1 \) is a trivial eigenvalue with right eigenvector \( \nu = [1, \ldots, 1]^T \neq 1_N \), and the first left eigenvector \( \gamma = [\gamma_i] \in \mathbb{R}^N \) satisfies \( A^T \gamma = \gamma \) [15].

B. Distributed Estimation of the First Left Eigenvector

Connectivity (and social standings in a connected network) of digraph \( D \) can be described by the first left eigenvector \( \gamma \) (and its components) [20]. Existing results on \( \gamma \) and its distributed estimation are briefly reviewed below [21].

The distributed observer consists of two parts: the first consists of an \( N \)th-order distributed observer; at the \( i \)th system, the observer state is defined by \( \theta^{(i)} = [\theta^{(i)}_1, \ldots, \theta^{(i)}_N]^T \in \mathbb{R}^N \), and evolution of the \( k \)th entry of \( \theta^{(i)} \) is defined by

\[
\dot{\theta}^{(i)}_k = \mu \sum_{j=1}^{N} a_{ij} (\theta^{(i)}_j - \theta^{(i)}_k) + \frac{\sin \omega t}{\sin t}, \quad \text{if } k = i
\]

\[
\dot{\theta}^{(i)}_k = \frac{\sin \omega t}{\sin t}, \quad \text{if } k \neq i
\]

where \( \mu \) is a constant gain, and integer \( \omega \in \mathbb{R}^+ \) is selected to be sufficiently large. As the second part, the estimate of the \( i \)th entry of \( \gamma \) is locally calculated at the \( i \)th system using the following formula:

\[
\hat{\gamma}_i = \lim_{\mu \to \infty} \frac{1}{\mu} \left[ I + \int_{-\Delta}^{\tau} \theta^{(i)}(\tau) \cos \omega t d\tau \right] \left[ I + \int_{-\Delta}^{\tau} \cos \omega t d\tau \right]^{-1}
\]

where \( \Delta \) is the period defined by \( \Delta = 2\kappa \pi/\omega \) with \( \kappa \in \mathbb{R}^+ \).
III. PROBLEM FORMULATION

In this paper, a DG refers to the combination of a PV (or another type of distributed energy resource) and a controllable load, e.g., electric vehicles, water heaters, and dump loads [4]. As shown in Fig. 1, the distribution network consists of up to $N$ DGs, the arrows denote the presence of local communication channels by which DGs can receive power dispatch command $P_D$ from the SCADA system or information from their neighboring DGs. A DG belongs to the leader set (denoted as $L$) if it receives $P_D$ from the SCADA system directly. Otherwise, it belongs to the follower set (denoted as $F$). At the $i$th DG, the instantaneous power generation capacity is denoted by $P_{MPP}^{(i)}$ (i.e., the PV operates in the MPPT mode and any power consumption by uncontrollable loads have already been taken out of $P_{MPP}^{(i)}$), its net power injection into the distribution network is $P_{DG}^{(i)}$, and the difference $(P_{MPP}^{(i)} - P_{DG}^{(i)})$ is absorbed by its controllable load or directly discarded by adjusting PV terminal voltage when the capacity of the controllable load is insufficient. Clearly, each DG can control its utilization ratio (or power injection ratio) which is defined by $\alpha^{(i)} \triangleq P_{DG}^{(i)}/P_{MPP}^{(i)}$.

It is obvious that $\alpha^{(i)} \in [0, 1]$. Should any of the DGs become inactive, $P_{MPP}^{(i)} = 0$ and $P_{DG}^{(i)} = 0$, the ratio is well defined to assume any chosen value between 0 and 1, which means that the proposed method meets the requirement of plug and play. Each of the DGs would exchange the information of its utilization ratio with its neighboring DGs. Through any strongly connected communication network, useful information gets implicitly propagated throughout the whole distributed network.

The objective of the proposed DCAGE design is twofold.

1) $P_{DG}^{(i)}$ is distributively controlled in such a way that the aggregated power dispatch is achieved, that is,

$$\sum_{i=1}^{N} P_{DG}^{(i)} = P_D. \quad (4)$$

2) Each of the DGs is treated equitably, i.e., they all operate with a common utilization ratio $\alpha^*$ as

$$\alpha^{(i)} \rightarrow \alpha^* \quad \forall i \in \{1, \ldots, N\} \quad (5)$$

where $\alpha^*$ is autonomously and adaptively found.

Should all the information be available globally, the power dispatch objective of (4) and (5) can be easily solved using a centralized control design since each DG can calculate

$$P_{\text{total}} \triangleq \sum_{i=1}^{N} P_{MPP}^{(i)}, \quad \alpha^* = \min \left\{ \frac{P_D}{P_{\text{total}}}, 1 \right\}. \quad (6)$$

Without global information, the total power generation capacity $P_{\text{total}}$ and consensus utilization ratio $\alpha^*$ are unknown. In what follows, a DCAGE approach is designed to attain the above objective following a similar idea to the centralized control.

IV. PROPOSED DCAGE DESIGN

The proposed DCAGE architecture consists of a distributed estimation algorithm for estimating $P_{\text{total}}$, a distributed control algorithm for controlling $P_{DG}$, and a distributed weights adjustment algorithm of the adjacency matrix. To enable the DGs with insufficient capacity of controllable load to operate in certain utilization ratio, a low-level DG power control strategy is also necessary. In the following, details on the above control blocks are to be elaborated.

A. Distributed Estimation of Aggregated Power Generation Capacity of the DGs

Since there is no global information available, $P_{\text{total}}$ is generally not known to each DG but can be estimated distributively and dynamically as: letting $P_{\text{tot,avr}}^{(i)}$ denote the estimate of the average of $P_{\text{total}}$ at the $i$th node and overtime interval $t \in [t_k, t_{k+1})$ (where $t_{k+1} - t_k = T_{\text{ini}}, T_{\text{ini}} = 1$ s in the simulation)

$$\dot{\hat{P}}_{\text{tot,avr}}^{(i)} = \mu \sum_{j=1}^{N} a_{ij}(t)[\hat{P}_{\text{tot,avr}}^{(j)}(t) - \hat{P}_{\text{tot,avr}}^{(i)}(t)] \quad (7)$$

where $\mu$ is a constant gain ($\mu = 100$ in the simulation). Weights $a_{ij}(t)$ should be deliberately chosen such that the resulting adjacency matrix $A(D)$ is doubly stochastic, which can be achieved with the distributed weights adjustment algorithm described in Section IV-C. The initial states in the time interval should satisfy $\hat{P}_{\text{tot,avr}}^{(i)}(t_k) = P_{MPP}^{(i)}(t_k)$.

Assumption 1: Assume that the communication network is strongly connected and the information of each node is available to itself, i.e., $a_{ij} > 0 \forall i \in \{1, \ldots, N\}$.

Theorem 1: Suppose that Assumption 1 holds and that adjacency matrix $A(D)$ is made to be doubly stochastic. Then, $\hat{P}_{\text{tot,avr}}^{(i)}$ converges to $P_{\text{total}}/N$ over the time interval $t \in [t_k, t_{k+1})$ by choosing gain $\mu$ large enough. That is, $P_{\text{total}}$ can be estimated by $N \cdot \hat{P}_{\text{tot,avr}}^{(i)}$.

Proof: Since $A(D)$ is doubly stochastic, the normalized left eigenvector $\boldsymbol{\gamma}$ corresponding to the eigenvalue 1 is $\mathbf{1}_N/N$, it follows from [15] that

$$\hat{P}_{\text{tot,avr}}^{(i)} \rightarrow \left[\hat{P}_{\text{tot,avr}}^{(i)}(t_k), \ldots, \hat{P}_{\text{tot,avr}}^{(N)}(t_k)\right] \gamma$$

$$= \sum_{i=1}^{N} P_{MPP}^{(i)}(t_k)/N \quad (8)$$

over the time interval $t \in [t_k, t_{k+1})$. ■
Remark 1: Because solar irradiance and the corresponding aggregated power generation capacity of the DGs are constantly varying, algorithm (7) should be initialized regularly to guarantee the estimate of $P_{total}$ remains up-to-date. Once $\hat{P}_{tot \ avr}^{(i)}$ is initialized in every time interval, it goes through its transient before reaching the consensus value. A sampling/holding link can be applied to resolve the problem, i.e., only the consensus value in each time interval is sampled and held as

$$\hat{P}_{tot \ avr}^{(i)}(t) = \begin{cases} 
0, & t \in [t_k, t_k + t_s) \\
k_y(P_{tot \ avr}^{(i)}(t) - \hat{P}_{tot \ avr}^{(i)}(t)) & t \in [t_k + t_s, t_{k+1} + t_s) 
\end{cases}$$

(9)

where $t_s \leq T_{ini}$ is long enough to guarantee that (7) can converge ($t_s = 0.2$ s in the simulation), and $k_y$ is a relatively large and positive gain. Therefore, $P_{tot \ avr}^{(i)}$ is almost piecewise constant over time.

B. Distributed Control of Utilization Ratios

The reference power $P_{DG}^{(i)}$ for each DG can be controlled by

$$P_{DG}^{(i)} = \alpha^{(i)} P_{MPP} \quad \forall i \in \{1, \ldots, N\}$$

(10)

where utilization ratio $\alpha^{(i)}$ evolves overtime interval $t \in [t_k + t_s, t_{k+1} + t_s)$ according to

$$\begin{cases} 
\alpha^{(i)} = \min \left\{ \frac{P_D}{N P_{tot \ avr}^{(i)}(t)}, 1 \right\} & \forall i \in \mathcal{L} \\
\alpha^{(i)} = \beta \sum_{j=1}^{N} a_{ij}(t) (\alpha^{(i)} - \alpha^{(j)}) & \forall i \in \mathcal{F} \end{cases}$$

(11)

where $\beta$ is a constant gain ($\beta = 300$ in the simulation). In the implementation of (11), the $i$th DG chooses its gains $a_{ij}$ based on the information received. As shown in [15] and [16], it is straightforward to initialize the row sum of $a_{ij}$ to be unity and, hence, adjacency matrix $A(D)$ row stochastic [as defined in (1)]. For instance, consider the simple choices for initial values of $a_{ij}$; for any choices of $\xi_{ij} > 0$

$$a_{ij}(0) = \frac{\xi_{ij} d_{ij}}{\sum_{l=1}^{N} \xi_{il} d_{il}}$$

(12)

where $d_{ij} = 1$ if node $j$ is adjacent to node $i$, and $d_{ij} = 0$ if otherwise.

Theorem 2: Consider the control law (11) and suppose that Assumption 1 holds. The utilization ratios of all the DGs converge uniformly and asymptotically to the consensus value $\alpha^*$ defined in (6).

Proof: It follows from [15, Th. 5.4] that, if the communication network is strongly connected, system (11) uniformly and asymptotically converges to $c 1_N$, where $c$ is a constant determined by the initial state and graph topology. It can be concluded that $c = \alpha^*$, since $\alpha^{(i)}(t) = \alpha^*, \forall i \in \mathcal{L}$. Hence,

$$\alpha = [\alpha^{(1)}, \ldots, \alpha^{(N)}]^T$$

uniformly and asymptotically converges to $\alpha^* 1_N$.

C. Distributed Algorithm of Making Adjacency Matrix Doubly Stochastic

For a strongly connected digraph, the adjacency matrix $A(D)$ can only be initialized to be row stochastic; thus, does not conform to Theorem 1’s requirement that $A(D)$ is doubly stochastic. It is known that any strongly connected digraph (satisfying Assumption 1) can be made doubly stochastic by invoking [23, Corollary 4.2]. In light of this, an online and distributed weights adjustment algorithm is proposed to adaptively make $A(D)$ also column stochastic. The proposed algorithm consists of two components: 1) It uses the distributed observer, introduced in Section II-B, to estimate the first left eigenvector $\gamma$. 2) Denoting sets $\arg \max \{\gamma_1, \ldots, \gamma_N\}$ and $\arg \min \{\gamma_1, \ldots, \gamma_N\}$ as $S_{max}$ and $S_{min}$, respectively, we adaptively adjust matrix $A(D)$ by varying the entries on the $i$th ($\forall i \in \{1, \ldots, N\}$) row according to the following adaptation laws:

Suppose that $\gamma_{max} = \max \{\gamma_1, \ldots, \gamma_N\}$ and $\gamma_{min} = \min \{\gamma_1, \ldots, \gamma_N\}$. Diagonal entry $a_{ii}$ ($\forall i \in \{1, \ldots, N\}$) evolves according to

$$\dot{a}_{ii} = \beta(1 - a_{ii}) \text{sign}(\gamma_{min} - \gamma_{max}),$$

if $i \in S_{max}$ and $a_{ii} > a_{min}$

$$\dot{a}_{ii} = \beta(1 - a_{ii}) \text{sign}(\gamma_{max} - \gamma_{min}),$$

if $i \in S_{min}$ and $a_{ii} > a_{min}$

$$\dot{a}_{ii} = 0,$$

if $i \notin S_{max} \cup S_{min}$ or $a_{ii} \leq a_{min}$

while nondiagonal entry $a_{ij}$ ($\forall i \in \{1, \ldots, N\}$ and $j \in N_i$) evolves according to

$$\dot{a}_{ij} = -\beta a_{ij} \text{sign}(\gamma_{min} - \gamma_{max}),$$

if $i \in S_{max}$ and $a_{ij} > a_{min}$

$$\dot{a}_{ij} = -\beta a_{ij} \text{sign}(\gamma_{max} - \gamma_{min}),$$

if $i \in S_{min}$ and $a_{ij} > a_{min}$

$$\dot{a}_{ij} = 0,$$

if $i \notin S_{max} \cup S_{min}$ or $a_{ij} \leq a_{min}$.

Initial conditions of $a_{ij}$ are chosen such that $A(D)$ is row stochastic and $a_{ii} \geq a_{min}, \beta > 0$ is a constant gain, and $a_{min} > 0$ is a predefined lower bound ($\beta = 0.3$ and $a_{min} = 0.05$ in the simulation).

In its implementation, the $i$th node only stores and computes entries of $a_{ij}$ in the $i$th row of $A(D)$; thus, the proposed algorithm is fully distributed. Properties of resulting matrix $A(D)$ are summarized into the following theorem.

Theorem 3: Suppose that Assumption 1 holds initially. Then, under control laws of (13) and (14), matrix $A$ has the following properties.

1) It remains to be nonnegative, row stochastic, and irreducible.

2) It converges to a column stochastic matrix.

Proof: The proof of Theorem 3 can be proceeded in two steps.

Step 1: It follows from (13) and (14) that, due to the presence of $a_{min}$, none of the entries of $A(D)$ will be negative and that, under (1), $\sum_{j=1}^{N} a_{ij} = 0, \forall i \in \{1, \ldots, N\}$. Therefore, matrix $A(D)$ remains to be row stochastic as long as it is initialized to be so. Also, since all the positive entries in the initial $A(D)$ will continue to be positive (as a result of $a_{min}$), matrix $A(D)$ remains irreducible.

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Step 2: Lemma 1 in the Appendix shows that under the control laws (13) and (14), \((\gamma_{\text{max}} - \gamma_{\text{min}})\) monotonically decreases to zero. Thus, \(A(D)\) will converge to a column stochastic matrix, which in turn leads to the conclusion of this theorem.

D. Low-Level DG Power Control Strategy

As \(P_{\text{DG}}^{(i)}\) can be calculated from (10), it is necessary for each DG to coordinate between the PV and controllable load, so that its aggregated power output can track \(P_{\text{DG}}^{(i)}\). Specifically, when the capacity of the \(i\)th controllable load \(P_{\text{CL max}}^{(i)}\) (if the controllable load is absent) is sufficient for the DG, its PV operates in the MPPT mode, with excess power \((P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)})\) being completely absorbed by the controllable load. As such \(P_{\text{MPP}}^{(i)}\) is the real power output of the PV. Otherwise, the controllable load operates at its full capacity (or the power consumption of the controllable load is zero if it is absent), while the PV operates below \(P_{\text{MPP}}^{(i)}\) by adjusting PV terminal voltage. The power reference for the PV \((P_{\text{PV ref}}^{(i)}\) and controllable load \((P_{\text{CL ref}}^{(i)}\) of the \(i\)th DG are expressed as

\[
P_{\text{PV ref}}^{(i)} = \begin{cases} 
P_{\text{MPP}}^{(i)}, & \text{if } P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)} \leq P_{\text{CL max}}^{(i)} \\
p_{\text{DG}}^{(i)} + P_{\text{CL max}}^{(i)}, & \text{if } P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)} > P_{\text{CL max}}^{(i)}
\end{cases}
\]

\[
P_{\text{CL ref}}^{(i)} = \begin{cases} 
P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)}, & \text{if } P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)} \leq P_{\text{CL max}}^{(i)} \\
P_{\text{CL max}}^{(i)}, & \text{if } P_{\text{MPP}}^{(i)} - P_{\text{DG}}^{(i)} > P_{\text{CL max}}^{(i)}
\end{cases}
\]

However, it should be noted that when the PV operates below \(P_{\text{MPP}}^{(i)}\), \(P_{\text{MPP}}^{(i)}\) is still required to initialize (7) and calculate (15) and (16), thus need to be simultaneously estimated. To this end, the quadratic interpolation-based PV power control method ([7], [13]) is improved to solve the dilemma.

The basic idea of the PV power control method in [7] is to use an iterative process to obtain the required voltage corresponding to \(P_{\text{PV ref}}^{(i)}\) by approximating the \(P-V\) characteristic curve with a quadratic curve.

Consider the \(k\)th iteration step in the algorithm of the \(i\)th PV, and assume that there are three sampling points known on the \(P-V\) characteristic curve, with their voltage and power values noted as \((U_{1}^{k}, P_{1}^{k})\), \((U_{2}^{k}, P_{2}^{k})\), and \((U_{3}^{k}, P_{3}^{k})\). Then, the following quadratic curve passing these points is used to approximate the \(P-V\) characteristic curve, as illustrated in Fig. 2(a):

\[
P_{\text{PV}}^{(i)}(U_{\text{PV}}^{(i)}) = aU_{\text{PV}}^{(i)} + bU_{\text{PV}}^{(i)} + c
\]

where \(P_{\text{PV}}^{(i)}\) is the power output of the \(i\)th PV corresponding to \(U_{\text{PV}}^{(i)}\), and the parameters \(a\), \(b\), and \(c\) are functions of \((U_{1}^{k}, P_{1}^{k})\), \((U_{2}^{k}, P_{2}^{k})\), and \((U_{3}^{k}, P_{3}^{k})\).

Equating \(P_{\text{PV}}^{(i)}(U_{\text{PV}}^{(i)})\) with \(P_{\text{PV ref}}^{(i)}\) derived in (15), the voltage corresponding to \(P_{\text{PV ref}}^{(i)}\) can be expressed as

\[
U_{\text{PV ref}}^{(i)} = \frac{-b - \sqrt{b^2 - 4a(c - P_{\text{PV ref}}^{(i)})}}{2a}
\]

(18)

By following properly designed iteration process [7], the real PV power output \(P_{\text{PV}}^{(i)}\) gradually converges to \(P_{\text{PV ref}}^{(i)}\).

Meanwhile, \(P_{\text{MPP}}^{(i)}\) can be estimated with the maximum value of the quadratic curve as

\[
P_{\text{MPP esti}}^{(i)} = -\frac{b}{2a}
\]

(19)

Define the relative error as

\[
\text{Error}\% = \frac{(P_{\text{MPP}}^{(i)} - P_{\text{MPP esti}}^{(i)})}{P_{\text{MPP}}^{(i)}} \times 100%.
\]

Fig. 2(b) shows by simulation that the relative error under various power output is satisfactorily small. Therefore, \(P_{\text{MPP esti}}^{(i)}\) can be used instead of \(P_{\text{MPP}}^{(i)}\) in (7), (15), and (16).

E. DCAGE Architecture

The architecture of the DCAGE control scheme is depicted in Fig. 3. The local control system at each DG communicates with its neighbors through the communication network. The structure of the local control system and its information flows are shown in the dash square of Fig. 3 (where, for the sake of clarity, only
the local control at DG \(i\) is shown in details). Specifically, the outcome of the distributed weights adjustment algorithm at DG \(i\) (i.e., \(a_{ij}, j \in \mathcal{N}_i\)) is sent to both distributed estimation of \(P_{\text{total}}\) and distributed control of utilization ratios. The current estimate of \(P_{\text{total}}\) is the input to the distributed power control algorithm so that \(\alpha^*\) can be set to \(\alpha^*\) if the DG belongs to set \(\mathcal{L}\). Through distributed control of utilization ratios, \(\alpha^*\) is propagated to DGs of set \(\mathcal{F}\) in the sense that utilization ratio of those DGs will converge to \(\alpha^*\). This distributed control scheme guarantees both (4) and (5).

V. SIMULATION RESULTS

To validate the effectiveness of the proposed DCAGE approach, numerical simulations are performed in the IEEE standard 34-bus distribution network. A six-node communication network is considered with it topology shown in Fig. 4, where the arrows denote possible information flow. Unless otherwise stated, node 1 is the only leader. It can be verified that the communication network is strongly connected.

In the subsequent case studies, five DGs are installed at nodes 1 to 5, with node 6 remaining inactive to demonstrate the flexibility of the proposed method. Each DG consists of a 0.3-MW PV and a 0.125-MW controllable load (while in practice, each DG may have different sizes of PV and controllable load). Initially, the irradiance at nodes 1 to 5 is set to 1000, 900, 800, 700, and 600 W/m\(^2\), respectively.

A. Test of the Weights Adjustment Algorithm

Based on the communication topology, adjacency matrix \(A(D)\) can be distributively initialized using (12) and the result is always row stochastic. The evolvement of the first left eigenvector and the diagonal entries in the adjacency matrix are shown in Fig. 5. Simulation results show that the estimate of \(\gamma\) at all the DGs are identical to Fig. 5(a), and the convergence performance of the estimator is prompt and accurate. Moreover, \(\gamma\) converges to \(1/6\), which indicates \(A(D)\) converges to a doubly stochastic matrix within 3 s. Hence the performance of the weights adjustment algorithm is satisfactorily fast. In the rest of the simulations, the matrix after convergence is used as the initial adjacency matrix.

B. Responses to Time-Varying Power Dispatch Command

In this case study, \(P_D\) changes from 1 to 0.25 MW at 0.5 s. The responses of the DGs to the change of \(P_D\) are depicted in Fig. 6. Fig. 6(a) and (b) indicates that \(\alpha^*\) decreases, and the aggregated power output of the DGs converge from 1 to 0.25 MW in about 2 s, which means the converge performance is comparable to the centralized control scheme. In Fig. 6(c), the estimates of \(P_{\text{MPP}}(i)\) at DG 1 to DG 4 deviate from their original values, suggesting that these DGs do not operate in the MPPT mode. Nevertheless, the estimates are accurate enough since they do not decrease too much. Fig. 6(d) compares the convergence performance under different number of leaders in the same setup. Not surprisingly, a communication network with two leaders (nodes 1 and 2) leads to a faster convergence rate to some extent than that with only one leader (node 1).

C. Dynamic Responses to Time-Varying Solar Irradiance

The power dispatch command remains 0.25 MW in this case. The solar irradiance of each DG decreases by 200 W/m\(^2\) at 1 s. The responses of the DGs to the change of solar irradiance are depicted in Fig. 7. An utilization ratio and aggregated power output dip can be seen in Fig. 7(a) and (b) right after solar irradiance decreases, since the terminal voltage of the PV cannot suddenly change. The power output of the PVs recovers soon by adjusting the terminal voltage with the method in Section IV-D. The estimate of power generation capacity of each DG is not precise in this transient, as can be seen in Fig. 7(c),
and consequently, the estimate of $P_{\text{total}}$ is not precise either from 1.2 to 2.2 s. When stability is reached in each DG, the estimate of power generation capacity of each DG is precise, but the estimate of total generation capacity is not precise until the algorithm in (7) is refreshed at 2.2 s. By far, the expected $\alpha^*$ can be acquired by the leader, and all the followers converge to it shortly afterward. In the steady state, all the PVs operate in the MPPT mode.

D. Robustness to Communication Interruptions

In what follows, it is verified that the proposed DCAGE approach is robust against communication interruptions. Specifically, communication channels from nodes 3 to 2 and from 5 to 3 malfunction at 1 s, and the communication network remains strongly connected thereafter. $P_D$ is 0.25 MW throughout the case. The responses of the system after communication interruptions are illustrated in Fig. 8. When a node detects the change of communication topology (i.e., some of its neighbors fail to send information to it), its corresponding row in the adjacency matrix $A$ is initialized [see Fig. 8(b)] and the distributed weights adjustment algorithm is activated. Fig. 8(c) and (d) shows that the left eigenvector changes suddenly and the estimate of total generation capacity also deviates from its real value. As a result, fluctuation in the output ratio of each DG can be observed in Fig. 8(a). By adjusting the weights, the adjacency matrix becomes doubly stochastic in about 3 s and all the DGs return to their original operational states.

VI. Conclusion

In this paper, a DCAGE approach is presented to dispatch multiple DGs in the distribution network. DGs consisting of PVs and controllable loads are considered in the problem formulation and strongly connected digraph is the sufficient condition for the communication topology. Among the DCAGE architecture, the distributed weights adjustment algorithm enables the adjacency matrix to become doubly stochastic, based on which the aggregated power generation capacity can be distributively estimated. Finally, the expected utilization ratio is obtained by the leaders and propagated to the followers with a consensus algorithm. The proposed control guarantees that each DG can adjust its power output, so that all of them operate in the same utilization ratio, while the aggregated power output can meet the power dispatch command from the SCADA system. Simulations in the IEEE benchmark distribution network validate the effectiveness and advantages of the proposed method. Furthermore, the proposed design methodology is also applicable to other problems, e.g., economic dispatch problem.

APPENDIX

Lemma 1: Consider nonnegative row stochastic matrix $A = [a_{ij}] \in \mathbf{R}^{N \times N}$ and its first left eigenvector $\gamma = [\gamma_i] \in \mathbf{R}^N$ with $\gamma_{\max} = \max\{\gamma_1, \ldots, \gamma_N\}$ and $\gamma_{\min} = \min\{\gamma_1, \ldots, \gamma_N\}$. As the entries of $A$ evolves according to (13) and (14), $(\gamma_{\max} - \gamma_{\min})$ monotonically decreases to 0.

Proof: Partition matrix $A$ as $A = [a_1, \ldots, a_N]^T$, where $a_i = [a_{i1}, \ldots, a_{iN}]^T$, $\forall i \in \{1, \ldots, N\}$. We define $S_{\min}^i = \{i | \gamma_i = \gamma_{\min}\}$ and $S_{\max}^i = a_{\max}^i \cup a_{\max}^{2i}$ where $S_{\max}^i = \{i | \gamma_i = \gamma_{\max}, a_{ij} > a_{im}\}$ and $S_{\max}^i = \{i | a_{ij} < \gamma_{\max}, a_{ij} = \gamma_{\min}\}$. Without loss of generality, it can be assumed that $S_{\max} = \{1, \ldots, r\}$, $S_{\min} = \{s, \ldots, N\}$, $S_{\max} = \{1, \ldots, u\} (u \leq r)$ and $S_{\max}^2 = \{u + 1, \ldots, r\}$, i.e., $\gamma_1 = \cdots = \gamma_r = \gamma_{\max}$ and $\gamma_s = \cdots = \gamma_N = \gamma_{\min}$.

It follows from control laws (13) and (14) that the derivative of the $i$th row of matrix $A$ satisfies

$$
\dot{a}_i = \left\{ \begin{array}{ll}
\beta a_i - \beta e_i & \forall i \in S_{\max}^1 \\
-\alpha a_i + \beta e_i & \forall i \in S_{\min} \\
0_N & \forall i \in \{1, \ldots, N\} - S_{\max}^1 - S_{\min}
\end{array} \right.
$$

(20)
where \( \mathbf{0}_N \in \mathbb{R}^N \) is an all-zero vector and \( \mathbf{e}_i \in \mathbb{R}^N \) is a vector of zeros except its \( i \)-th entry being one.

As such, the derivative of \( A^T \) can be written in a compact form as

\[
\frac{dA^T}{dt} = [a_1, \ldots, a_N] \Lambda - [e_1, \ldots, e_N] \Lambda = (A^T - I_N) \Lambda
\]

where \( I_N \in \mathbb{R}^{N \times N} \) is the identity matrix, \( \Lambda \triangleq \text{diag}((\beta I_q, O_{s-u-1} \rightarrow \beta I_{N-s+1}), \text{and } O_{s-u-1} \in \mathbb{R}^{(s-u-1) \times (s-u-1)} \) is an all-zero matrix.

For the simplicity of the proof, \( \gamma \) is scaled so that \( \gamma_{\max} = 1 \). Since there is \( A^T \gamma = \gamma \), its derivative can be expressed as

\[
(I_N - A^T) \frac{d\gamma}{dt} = \frac{dA^T}{dt} \gamma, \quad \gamma_{\max} = 1.
\]

Substituting (21) into (22), there is

\[
(I_N - A^T) \left( \frac{d\gamma}{dt} + \Lambda \gamma \right) = 0, \quad \gamma_{\max} = 1.
\]

Since the digraph is strongly connected, matrix \((I_N - A^T)\) has a simple eigenvalue zero to \([15, \text{Th. 4.8}]\). Hence, \( \text{Null}(I_N - A^T) = \text{span}\{\gamma\} \), and

\[
\frac{d\gamma}{dt} = (\alpha I_N - \Lambda) \gamma,
\]

where \( \alpha \) is a variable determined by \( \gamma_{\max} = 1 \) and to be analyzed later.

Next, we calculate the derivative of \( \gamma_{\min} \) according to the above equation and consider two cases: 1) \( S_{\max}^2 = \emptyset \); and 2) \( S_{\max}^2 \neq \emptyset \).

For the first case, \( \forall i \in S_{\max}^1 \), since \( \gamma_i = \gamma_{\max} = 1 \), there is \( \frac{d\gamma_i}{dt} = 0 \). It follows from (24) that

\[
\frac{d\gamma_i}{dt} = \alpha \gamma_i - \Lambda_{ii} \gamma_i = (\alpha - \beta) \gamma_i = 0 \quad \forall i \in S_{\max}^1
\]

i.e., \( \alpha = \beta \). Consequently, (24) implies that

\[
\frac{d\gamma_i}{dt} = \alpha \gamma_i - \Lambda_{ii} \gamma_i = 2\beta \gamma_i > 0 \quad \forall i \in S_{\min}^1
\]

Similarly, for the second case, there is

\[
\frac{d\gamma_i}{dt} = \alpha \gamma_i - \Lambda_{ii} \gamma_i = \alpha \gamma_i = 0 \quad \forall i \in S_{\max}^2
\]

which implies that \( \alpha = 0 \). As a result, it follows from (24) that

\[
\frac{d\gamma_i}{dt} = \alpha \gamma_i - \Lambda_{ii} \gamma_i = -\beta \gamma_i < 0 \quad \forall i \in S_{\max}^1
\]

and

\[
\frac{d\gamma_i}{dt} = \alpha \gamma_i - \Lambda_{ii} \gamma_i = \beta \gamma_i > 0 \quad \forall i \in S_{\min}^1
\]

Note that (28) indicates that when \( S_{\max}^2 \neq \emptyset \), \( \gamma_i \) begins to decrease and no longer belongs to \( S_{\max}^1 \), but \( \gamma_i = 1 \) \((\forall i \in S_{\max}^1) \) remains in \( S_{\max}^1 \). Therefore, (27) is still valid.

In summary, it follows from (26) and (29) that by fixing \( \gamma_{\max}^* \), \( \gamma_{\min} \) is monotonically increasing until \( (\gamma_{\max}^* - \gamma_{\min}) \) converges to 0.
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